Adaptive Methods for Optimization in Stochastic Environments

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February 2, 2018

CRiSTAL & Inria Lille

Multi-armed Bandit

Multi-armed Bandit Game

- What is a MAB game?

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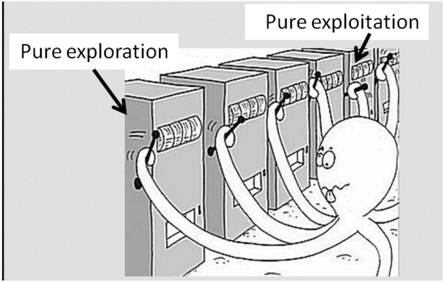
- Objective: maximize the total reward



Source: Microsoft Research

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- However, this does not seem to be always the right way to base the strategies on in some scenarios...

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Current State-of-the-Art

- Finitely-armed algorithms: Successive Reject [Audibert et al. 2010], Sequential Halving [Karnin et al. 2013], UGapE [Gabillon et al. 2013]...
- Infinitely-armed algorithms: SiRI [Carpentier and Valko 2015], Hyperband [Li et al. 2017]

Black Box Optimization and Beyond...

Reformulation in the context of Optimization

- An unknown noisy function $f : \mathcal{X} \to \mathbb{R}$.
- At each step t, a policy picks an action $\mathsf{x_t} \in \mathcal{X}$ and receives a reward $r_t = f(\mathsf{x_t}) + \epsilon_\mathsf{t}$ where ϵ_t is the noise.
- **Simple regret:**

$$S_n = f(\mathsf{x}^*) - f(\mathsf{x}_{j_n}).$$

- **Cumulative regret:**

$$R_n = \sum_{1 \leq t \leq n} (f(\mathsf{x}^*) - f(\mathsf{x}_t)).$$

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- **Hierarchical Optimization**: HOO [Bubeck et al. 2011], POO [Grill et al. 2015], HCT Gheshlaghi-Azar et al. 2014]...
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- **Perspective**:
    - New best arm identification algorithms based on hierarchical exploration?
    - Adaptive partitioning?
    - Anytime?

Hyperparameter Optimization

Experiment

- select a set of hyper-parameters $\mathbf{x}_t$ as an arm
- $\mathbf{x}_t$ is then used in some machine learning classifier
- recommend an arm $\mathbf{x}_{j_t}$

- **Loss function**:
  - Logistic loss for classification problems
  - Mean squared error for regression problems
- The underlying task is to find some classifier $g_{\mathbf{x}_t}$ which minimizes the expected loss $f(g_{\mathbf{x}_t}) = \mathbb{E}\left[\mathcal{L}(\boldsymbol{y}, g_{\mathbf{x}_t}(X)\right]$

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- **Output:** the best intermediate loss ever seen

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- Pros: strong anytime performance, easily parallelizable
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- **Perspective:** take into account previously sampled configurations? $\rightarrow$ TPE+Hyperband [Falkner et al. 2017]

Contextual Bandits and Algorithm Selection

Contextual Bandits

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- A policy $\pi \in \Pi = \{\pi : C \to 1 \cdots K\}$ chooses one arm k_t to play
- We want to minimize the (cumulative) regret:

$$R_n = \max_{\pi \in \Pi} \sum_{1 \leq t \leq n} r_t(\pi(c_t)) - \sum_{1 \leq t \leq n} r_t(k_t)$$

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- Can be seen as a contextual bandit problem
- **Perspective:** LinUCB [Li et al. 2010]? Comparable to greedy approach?

Thank you!